Quantized charge pumping in superconducting double barrier structure : Non-trivial correlations due to proximity effect

Arijit Saha*
Harish-Chandra Research Institute,
Chhatnag Road, Jhusi, Allahabad 211019, India

Sourin Das†
Department of Condensed Matter Physics,
Weizmann Institute of Science, Rehovot 76100, Israel

(Dated: August 12, 2008)

We consider quantum charge pumping of electrons across a superconducting double barrier structure in the adiabatic limit. The superconducting barriers are assumed to be reflection-less so that an incident electron on the barrier can either tunnel through it or Andreev reflect from it. In this structure, quantum charge pumping can be achieved *(a)* by modulating the amplitudes, Δ_1 and Δ_2, of the gaps associated with the two superconductors or alternatively, *(b)* by a periodic modulation of the order parameter phases, ϕ_1 and ϕ_2 of the superconducting barriers. In the former case, we show that the superconducting gap gives rise to a very sharp resonance in the transmission resulting in quantization of pumped charge, when the pumping contour encloses the resonance. On the other hand, we find that quantization is hard to achieve in the latter case. We show that inclusion of weak electron-electron interaction in the quantum wire leads to renormalisation group evolution of the transmission amplitude towards the perfectly transmitting limit due to interplay of electron-electron interaction and proximity effects in the wire. Hence as we approach the zero temperature limit, due to renormalisation group flow of transmission amplitude we get destruction of quantized pumped charge. This is in sharp contrast to the case of charge pumping in a double barrier through a Luttinger liquid where quantized charge pumping is actually achieved in the zero temperature limit.

PACS numbers: 73.23.-b,74.78.Na,74.45.+c

I. INTRODUCTION

The phenomena of quantum charge pumping corresponds to a net flow of DC current between different electron reservoirs (at zero bias) connected via a quantum system whose system parameters are periodically modulated in time[1,2]. The zero bias current is obtained in response to the time variation of the parameters of the quantum system which explicitly break the time reversal symmetry. It is worth mentioning that breaking of the time-reversal symmetry is necessary but not a sufficient condition in order to get net pumped charge in unit cycle. For obtaining a net pumped charge, parity or spatial symmetry must also be broken. Within a scattering approach, if the time period of modulation of the scattering system parameters is much larger than the time the particle spends inside the scattering region, adiabatic limit is reached. In this limit, the pumped charge in a unit cycle becomes independent of the pumping frequency. This is referred to as "adiabatic charge pumping". Experimentally charge pumping has been observed in mesoscopic systems involving quantum dots and carbon nanotubes[3,4,5]. Ofcourse one has to be very careful in interpreting the experimentally observed pumped charge as it can be faked by rectification effects as was pointed out by Brouwer[6].

In the recent years, there has been an upsurge of research interest in exploring the effects due to inclusion of electron-electron ($\mathsf{e}-\mathsf{e}$) interaction on the pumped charge[6,7,8,9,10,11,12]. In this article, we explore the effect of inter-electron interaction on the charge pumped across a superconducting double barrier (SDB) system[13] in the context of one-dimensional (1–D) quantum wire (QW). Pumping of free electrons across 1–D quantum well was studied earlier in Refs. 14,15,16, where using Brouwer's formula[1], it was shown that the pumped charge can be expressed as a sum of two contributions, viz., a dissipative part and a quantized topological part, the latter being independent of the details of the pumping contour[17,18]. The dissipative part was found to be proportional to the conductance through the system on the pumping contour in the parameter space while the topological part was non-zero only if the pumping contour enclosed a resonance. Hence in order to obtain quantized pumped charge, one needs to reduce the dissipative part as much as possible. This is very naturally achieved if one considers pumping through a quantum well in a 1–D interacting electron gas[7,19] (Luttinger liquid) as in this case interaction correlations make the resonance very sharp thereby reducing the conductance on the contour enclosing the resonance to vanishingly small values in the zero temperature limit. This leaves behind a quantized topological part. The pumped charge was shown to converge to a quantized value asymptotically. This was obtained using a perturbative approach for the case of a weakly interacting electron gas followed by "Poor-man's scaling approach"[20]. In this article, we show that the presence of inter-electron interaction in

the wire leads to nontrivial scattering processes due to proximity effects which leads to a power law reduction of the pumped charge from the quantized value (as opposed to enhancement) in the adiabatic limit as one lowers the temperature. Quantum charge pumping using various setups involving superconductor has been a topic of major interest in recent past[21,22,23,24,25,26,27,28,29,30]. Specifically we consider pumping of electrons (in the adiabatic limit) across a SDB structure, as depicted in Fig. 1. Till date no experiment has been carried out in the context of charge pumping for the case of superconducting barrier. Experimentally it might be possible to design a SDB structure by depositing thin strips of superconducting material on top of a single ballistic QW (like carbon nanotubes) at two places, which can induce a finite superconducting gap in the barrier regions of the QW as a result of proximity of the superconducting strips. In our simple-minded theoretical modelling of the system we assume that the superconducting barrier (SB) to be reflection-less so that an incident electron on the barrier can either tunnel through it or Andreev reflect from it. Within the simplified theoretical model, we explore two scenarios to achieve quantization of pumped charge − *(a)* by periodic modulation of amplitudes Δ_1 and Δ_2 of the gap at the two SB or alternatively, *(b)* by periodic modulation of the order parameter phases ϕ_1 and ϕ_2 associated with the two SB. For free electrons in the QW we show that in the $\Delta_1 - \Delta_2$ plane, there is an isolated sharp resonance point in transmission probability across the SDB structure. On the other hand transmission probability across the double barrier has a line of sharp resonances in the $\phi_1 - \phi_2$ plane. As mentioned earlier, in order to obtain quantized pumped charge, the transmission on the pumping contour should be as small as possible. When we consider Δ_1 and Δ_2 as the pumping parameters, we can always choose a pumping contour which completely encloses the isolated resonance and hence it is possible to achieve quantization of charge if the resonance is sharp enough. However, in the $\phi_1 - \phi_2$ plane, we have a line of resonances. Any closed contour enclosing the resonances will surely cross the resonance line at least twice thereby increasing the dissipative part and consequently resulting in destruction of quantization of pumped charge. Interestingly enough, inclusion of weak e − e interaction in the wire results in a RG flow of the transmission amplitude towards perfectly transmitting limit due to proximity induced effect on the interacting electrons in the QW as we lower the energy scale such as temperature. Hence the sharpness of resonance is lost due to RG enhancement of transmission through individual SB resulting in complete destruction of quantized charge pumping as we go down in temperature. It is worth noticing that the consequence of inclusion of correlations due to e − e interaction is just opposite here with respect to the case of double barrier in a Luttinger liquid[19].
This article is organised as follows. In Sec. II, we discuss the modelling of SDB in a 1–D QW and calculate the transmission and Andreev reflection (AR) amplitudes of the system. In Sec. III, we discuss the renormalisation group (RG) flow for transmisson and AR for the SB. In Sec. IV, we discuss our RG scheme for the SDB and calculate the pumped charge. In the end, we discuss our results and give conclusions in Sec. V.

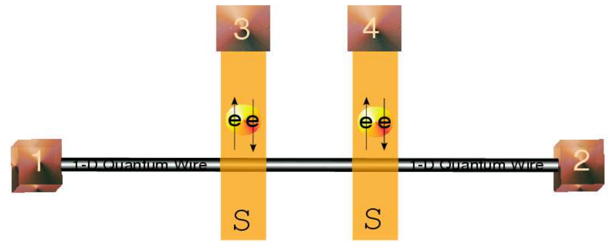

FIG. 1: A one-dimensional quantum wire (*e.g.* carbon nanotube) connected to two reservoirs, labelled by 1 and 2. The two thin strips on the wire depict layers of superconducting material deposited on top of the wire. The superconducting strips are connected to contacts labelled 3 and 4.

II. SUPERCONDUCTING BARRIER

Quantum transport in SB structure was considered in past in Ref. 13. Here we consider a very similar set-up comprising of a ballistic 1–D QW with two short, but finite superconducting patches as shown in Fig. 1. Here, $\Delta^{(i)}$ is the pair potential on the two patches (i refers to the index of the strip). Following Ref. 13, we use the Bogolubov−de Gennes (BdG) equation[31,32] to calculate the transmission amplitude $t^{(i)}_{ee}$ and the AR amplitude, $r^{(i)}_{eh}$, where i is the barrier index. The space dependance of the order parameter (which also acts as the scattering potential) for the incident electron can be expressed as

$$V(x) = \Delta^{(i)} e^{i\phi_1} \Theta(x)\Theta(-x+a) + \Delta^{(i)} e^{i\phi_2} \Theta[x-(a+L)]\Theta[-x+(2a+L)] \qquad (1)$$

where, a is the width of the SB and L is the distance between the two barriers.
Hence the BdG equations can be written as,

$$Eu_+ = \left[\frac{-\hbar^2\nabla^2}{2m} + V(x) - \mu\right] u_+ + \Delta u_- \qquad (2)$$

$$Eu_- = \left[\frac{\hbar^2\nabla^2}{2m} - V(x) + \mu\right] u_- + \Delta^\star u_+ \qquad (3)$$

Solving the BdG equation in the normal and superconducting regions and matching the solution at $x = 0$ and $x = a$, we get

$$t^{(i)}_{ee} = \frac{e^{ik^+a}(u_+^2 - u_-^2)}{u_+^2 - u_-^2 e^{i(k^+-k^-)a}}\,; \quad t^{(i)}_{hh} = \frac{e^{-ik^-a}(u_+^2 - u_-^2)}{u_+^2 - u_-^2 e^{i(k^+-k^-)a}}$$

$$r_{eh}^{(i)} = \frac{u_+ u_- e^{-i\phi_i}(1 - e^{i(k^+ - k^-)a})}{u_+^2 - u_-^2 e^{i(k^+ - k^-)a}}$$
$$r_{he}^{(i)} = \frac{u_+ u_- e^{i\phi_i}(1 - e^{i(k^+ - k^-)a})}{u_+^2 - u_-^2 e^{i(k^+ - k^-)a}} \qquad (4)$$

where $\hbar k^{\pm} = \sqrt{2m(E_F \pm (E^2 - \Delta^{2(i)})^{1/2})}$, $u_{\pm} = \frac{1}{\sqrt{2}}[(1 \pm (1 - (\Delta^{(i)}/E)^2)^{1/2})]^{1/2}$ Here, $t_{ee}^{(i)}$, $t_{hh}^{(i)}$, $r_{eh}^{(i)}$, $r_{he}^{(i)}$ are the transmission and AR amplitudes. m is the effective mass of the electron in the wire, E_F is the Fermi energy for the electrons in the superconducting region, and E is the Fermi energy of electrons in the normal region of the QW, measured with respect to E_F. Hence the scattering matrix for the single SB problem for an incident electron or hole is given by

$$S_e = \begin{vmatrix} r_{eh}^{(i)} & t_{ee}^{(i)} \\ t_{ee}^{(i)} & r_{eh}^{(i)} \end{vmatrix} \quad \text{and} \quad S_h = \begin{vmatrix} r_{he}^{(i)} & t_{hh}^{(i)} \\ t_{hh}^{(i)} & r_{he}^{(i)} \end{vmatrix} \qquad (5)$$

Using the S-matrix given by Eq. 5, we obtain the effective S-matrix for the double barrier system[33]. We assume particle-hole symmetry, hence $t_{ee} = t_{hh}$ and $r_{eh} = r_{he}$. The net transmission and net AR amplitude through the double barrier are

$$T_{ee} = \frac{t_{ee}^{(1)} t_{ee}^{(2)} e^{iq^+ L}}{1 - r_{eh}^{(2)} r_{he}^{(1)} e^{i(q^+ - q^-)L}}$$
$$R_{eh} = r_{eh}^{(1)} + \frac{t_{ee}^{(1)} r_{eh}^{(2)} t_{hh}^{(1)} e^{i(q^+ - q^-)L}}{1 - r_{eh}^{(2)} r_{he}^{(1)} e^{i(q^+ - q^-)L}} \qquad (6)$$

where $\hbar q^{\pm} = \sqrt{2m(E_F \pm E)}$. In order to obtain quantization of pumped charge, we choose to operate in the sub-gap regime *i.e.*, $E < \Delta$. In this regime, $|T_{ee}|^2$ has sharp resonances at discrete values of E/Δ for a given value of $\phi_1 - \phi_2$[13]. These resonances result from multiple AR of electron to hole and vice-versa inside the double barrier.

III. WIRG STUDY OF JUNCTIONS

We study the effects of inter-electron interactions in the wire on the S-matrix characterizing the superconducting barrier using the RG method introduced in Ref. 20, and the generalizations to multiple wires in Refs. 34,35. The basic idea of the method is as follows. The presence of back-scattering (reflection) induces Friedel oscillations in the density of non-interacting electrons. Within a mean field picture for weakly interacting electron gas, the electron not only scatters off the potential barrier but also scatters off these density oscillations with an amplitude proportional to the interaction strength. Hence by calculating the total reflection amplitude due to scattering from the scalar scatterer and from the Friedel oscillations created by the scatterer, we can include the effect of $\mathsf{e-e}$ interaction in calculating transport. This can now be generalized in a similar spirit to the case where there is, besides non-zero reflection also non-zero AR which turns an incoming electron into an outgoing hole due to proximity effects as done in Ref. 36 and then generalized to multiple wire superconducting junction in Ref. 37.

The fermion field on each wire can be written as,

$$\psi_{is}(x) = \Psi_{I\,is}(x)\, e^{i\,k_F\,x} + \Psi_{O\,is}(x)\, e^{-i\,k_F\,x} \qquad (7)$$

where i is the wire index, s is the spin index which can be $\uparrow, \downarrow$ and I, O stands for outgoing or incoming fields. Note that $\Psi_I(x)(\Psi_O(x))$ are slowly varying fields on the scale of k_F^{-1} and contain the annihilation operators as well as the slowly varying wave-functions. The expectation values for the density $\langle\, \Psi_{is}^{\dagger} \Psi_{is}\, \rangle$ gives (dropping the wire index),

$$\langle\, \psi_{O\uparrow}^{\dagger} \psi_{I\uparrow}\, \rangle = \langle\, \psi_{O\downarrow}^{\dagger} \psi_{I\downarrow}\, \rangle = \frac{i\, r^{\star}}{4\pi x} \qquad (8)$$
$$\text{and} \quad \langle\, \psi_{I\uparrow}^{\dagger} \psi_{O\uparrow}\, \rangle = \langle\, \psi_{I\downarrow}^{\dagger} \psi_{O\downarrow}\, \rangle = \frac{-i\, r}{4\pi x}. \qquad (9)$$

Hence, besides the density, the expectation values for the pair amplitude $\langle\, \Psi_{is}^{\dagger} \Psi_{is}^{\dagger}\, \rangle$ and its complex conjugate $\langle\, \Psi_{is} \Psi_{is}\, \rangle$ are also non-zero and are given by (dropping the wire index)

$$\langle\, \psi_{O\uparrow}^{\dagger} \psi_{I\downarrow}^{\dagger}\, \rangle = -\langle\, \psi_{O\downarrow}^{\dagger} \psi_{I\uparrow}^{\dagger}\, \rangle = \frac{-i\, r_A}{4\pi x} \qquad (10)$$
$$\text{and} \quad \langle\, \psi_{O\uparrow} \psi_{I\downarrow}\, \rangle = -\langle\, \psi_{O\downarrow} \psi_{I\uparrow}\, \rangle = \frac{-i\, r_A^{\star}}{4\pi x}. \qquad (11)$$

So, we see that the Boguliobov amplitudes fall off as $1/x$ just like the normal density amplitudes.
We now allow for short-range density-density interactions between the fermions

$$\mathcal{H}_{\mathsf{int}} = \frac{1}{2} \int dx\, dy \left(\sum_{s=\uparrow,\downarrow} \rho_s \right) V(x-y) \left(\sum_{s=\uparrow,\downarrow} \rho_s \right) \qquad (12)$$

to obtain the standard four-fermion interaction Hamiltonian for spin-full fermions as

$$\begin{aligned} \mathcal{H}_{\mathsf{int}} = \int dx \Big[& g_1 \Big(\Psi_{I\uparrow}^{\dagger} \Psi_{O\uparrow}^{\dagger} \Psi_{I\uparrow} \Psi_{O\uparrow} + \Psi_{I\downarrow}^{\dagger} \Psi_{O\downarrow}^{\dagger} \Psi_{I\downarrow} \Psi_{O\downarrow} \\ & + \Psi_{I\uparrow}^{\dagger} \Psi_{O\downarrow}^{\dagger} \Psi_{I\downarrow} \Psi_{O\uparrow} + \Psi_{I\downarrow}^{\dagger} \Psi_{O\uparrow}^{\dagger} \Psi_{I\uparrow} \Psi_{O\downarrow} \Big) \\ & + g_2 \Big(\Psi_{I\uparrow}^{\dagger} \Psi_{O\uparrow}^{\dagger} \Psi_{O\uparrow} \Psi_{I\uparrow} + \Psi_{I\downarrow}^{\dagger} \Psi_{O\downarrow}^{\dagger} \Psi_{O\downarrow} \Psi_{I\downarrow} \\ & + \Psi_{I\uparrow}^{\dagger} \Psi_{O\downarrow}^{\dagger} \Psi_{O\downarrow} \Psi_{I\uparrow} + \Psi_{I\downarrow}^{\dagger} \Psi_{O\uparrow}^{\dagger} \Psi_{O\uparrow} \Psi_{I\downarrow} \Big) \Big] \end{aligned} \qquad (13)$$

where g_1 and g_2 are the interaction parameters[38].

The effective Hamiltonian can be derived using a Hartree−Fock (HF) decomposition of the interaction Hamiltonian. The charge conserving HF decomposition

leads to the interaction Hamiltonian (normal) of the following form on each half wire,

$$\mathcal{H}^N_{\mathsf{int}} = \frac{-i(g_2-2g_1)}{4\pi}\int_0^\infty \frac{dx}{x}\Big[r^\star\left(\Psi^\dagger_{I\uparrow}\Psi_{O\uparrow}+\Psi^\dagger_{I\downarrow}\Psi_{O\downarrow}\right) - r\left(\Psi^\dagger_{O\uparrow}\Psi_{I\uparrow}+\Psi^\dagger_{O\downarrow}\Psi_{I\downarrow}\right)\Big] \quad (14)$$

(We have assumed spin-symmetry *i.e.* $r_\uparrow = r_\downarrow = r$.) This has been derived earlier[34]. Using the same method, but now also allowing for a charge non-conserving HF decomposition we get the (Andreev) Hamiltonian

$$\mathcal{H}^A_{\mathsf{int}} = \frac{-i(g_1+g_2)}{4\pi}\int_0^\infty \frac{dx}{x}\Big[-r_A^\star(\Psi^\dagger_{I\uparrow}\Psi^\dagger_{O\downarrow}+\Psi^\dagger_{O\uparrow}\Psi^\dagger_{I\downarrow}) + r_A\left(\Psi_{O\downarrow}\Psi_{I\uparrow}+\Psi_{I\downarrow}\Psi_{O\uparrow}\right)\Big] \quad (15)$$

The $\mathsf{e}-\mathsf{e}$ interaction induced amplitude to go from an incoming electron wave to an outgoing electron wave under $e^{-i\mathcal{H}^N_{\mathsf{int}}t}$ (for electrons with spin) is given by[34]

$$\frac{-\alpha\, r_s}{2}\,\ln(kd) \quad (16)$$

where $\alpha = (g_2-2g_1)/2\pi\hbar v_F$ and d is the short distance cut-off for the RG flow. Analogously, the amplitude to go from an incoming electron wave to an outgoing hole wave under $e^{-i\mathcal{H}^A_{\mathsf{int}}t}$ is given by[36]

$$\frac{\alpha'\, r_A}{2}\,\ln(kd) \quad (17)$$

where $\alpha' = (g_1+g_2)/2\pi\hbar v_F$.
These logarithmic corrections to the bare reflection amplitude and the AR amplitude can be summed up using a “Poor-man’s scaling approach”[39] which finally leads a RG equation for r and r_A.

IV. RENORMALISATION GROUP SCHEME AND THE PUMPING FORMULA

We include the effects due to proximity of superconductor and $\mathsf{e}-\mathsf{e}$ interaction in the wire via a RG approach developed very recently[37] for the case of 1–D normal metal−superconductor−normal metal (NSN) junction. As we are only interested in the coherent regime ($L_T >> L$, L_T is the thermal length), we can effectively treat the SDB system (NSNSN junction) as a single barrier (NSN junction) as far as RG is concerned.

Hence the effective two-channel S-matrix for this double barrier system can be written as

$$S = \begin{vmatrix} |R_{eh}|e^{i\theta} & |T_{ee}|e^{i\phi} \\ |T_{ee}|e^{i\phi} & |R_{eh}|e^{i\theta'} \end{vmatrix} \quad (18)$$

where all the amplitudes and phases associated with the matrix elements are functions of the time-varying parameters, $V_i(t) = V_0 + P\cos(\omega t + (-1)^{i-1}\eta)$ where $i = 1,2$ stands for the barrier index. $V_i = \Delta_i$ and $V_i = \phi_i$ are the two possible pumping parameters. The reflection coefficients are not the same (phases can differ) because the time-varying potentials explicitly violate parity. In principle the S-matrix also violates time-reversal invariance. But since in the adiabatic approximation, we are only interested in instantaneous hamiltonian. Note that the instantaneous S-matrix can mimic a time-reversal symmetric S-matrix.

Using the modified Brouwer’s formula[30], the pumped charge can directly be obtained from the parametric derivatives of the S-matrix elements. It is worth mentioning that even though Brouwer’s formula is valid for non-interacting electron system, we are able to use it here because effects due to interactions in the wires can be taken care of by the renormalization of the bare S-matrix obtained for the free-electron case.

For single channel S-matrix, we have

$$\mathcal{Q} = \frac{e}{2\pi}\int_0^\tau dt\ \mathrm{Im}\left[-\frac{\partial S_{11}}{\partial V_1}S^\star_{11}\dot V_1 + \frac{\partial S_{12}}{\partial V_1}S^\star_{12}\dot V_1 - \frac{\partial S_{11}}{\partial V_2}S^\star_{11}\dot V_2 + \frac{\partial S_{12}}{\partial V_2}S^\star_{12}\dot V_2\right] \quad (19)$$

where S_{ij} denote the elements of the S-matrix. Note the negative sign in the above expression, which results from the fact that S_{11} corresponds to conversion of an electron into a hole. Thus, the pumped charge is directly related to the amplitudes and phases that appear in the S-matrix. Inserting Eq. 18 in Eq. 19,

$$\mathcal{Q} = \frac{e}{2\pi}\int_0^\tau \Big[\,\dot\theta - G(t)(\dot\theta+\dot\phi)\,\Big]\,dt \quad (20)$$

Here $G(t) = |T_{ee}(t)|^2$ is the instantaneous two terminal linear conductance (labelled by $1,2$ in Fig. 1), in units of $2e^2/h$. The first term on the RHS in Eq. 20 is clearly quantized since $e^{i\theta}$ returns to itself at the end of one cycle. So the only possible change in θ in a period can be in integral multiples of 2π *i.e.*, $\theta(\tau) \to \theta(0) + 2\pi n$ ($n \to$ integer). The second term is the ‘dissipative’ term which prevents the perfect quantization. The second term is directly proportional to the two terminal Landauer−Buttiker conductance for the system on the pumping contour. The relative sign between $\dot\theta$ and $\dot\phi$ in the expression for pumped charge in Eq. 20 originates from the AR process, which converts an electron to a hole. This is in contrast to what has been found for the normal double barrier problem[19]. For a reflectionless junction, the basic idea of the RG method is as follows. The presence of a superconductor induces a finite yet weak pair potential in the QW resulting in scattering of incoming electrons to outgoing holes (Andreev processes) in the wire, away from the junction. Hence by calculating the total AR amplitude, due to scattering from the NSN junction and the (weak) pair potential in the wire perturbatively in interaction strength and followed by “Poor-man’s scaling” approach, we obtain the

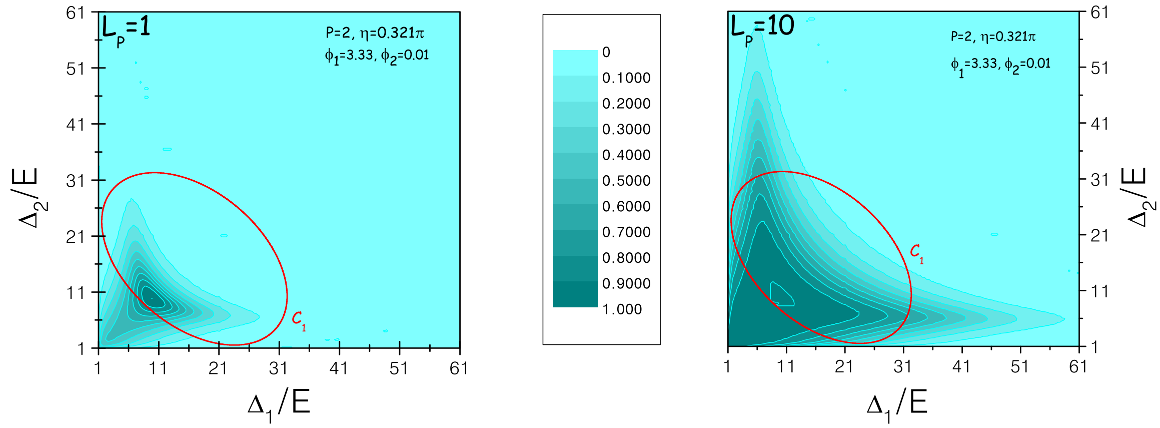

FIG. 2: Contours of transmission probability, $|T_{ee}|^2$ in the $\Delta_1 - \Delta_2$ plane at two different values of length scale, $L_P = 1$ and $L_P = 10$, at which the RG flow is cut-off for values of $V(0) = 0.1$ and $V(2k_F) = 0.1$. The red ellipse C_1 represents the pumping contour.

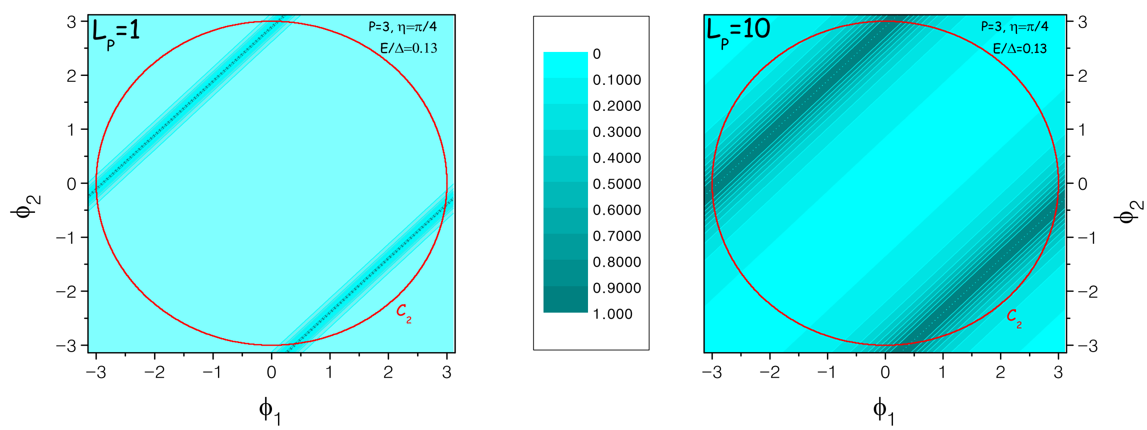

FIG. 3: Contours of transmission probability, $|T_{ee}|^2$ in the $\phi_1 - \phi_2$ plane at two different values of length scale, $L_P = 1$ and $L_P = 10$ at which the RG flow is cut-off for values of $V(0) = 0.1$ and $V(2k_F) = 0.1$. The red circle C_2 represents the pumping contour.

RG equation for the elements of the effective S-matrix of the SDB structure in the coherent regime ($L_T > L$). So, the entries of S-matrix therefore become functions of the length scale L_P due to the RG flow. The RG flow can also be considered to be a flow in the temperature since the length scale L_P can be converted to a temperature scale using the thermal length $L_T = \hbar v_F/(k_B T)$. Hence, the RG flow has to be cut-off by either L_T, or the system size L_S, whichever is smaller[35].

Without loss of generality, we can calculate the renormalized S-matrix at different length scales or equivalently at different temperatures at any point on the pumping contour. Hence, to avoid unnecessary complications arising due to the RG flow of phases associated with S-matrix elements (θ, θ', ϕ), we choose to calculate the RG flow of the S-matrix when the barriers are symmetric. This symmetry leads to vanishing of the RG flow of the phases hence making the calculation algebrically simple.

The RG flow of the normal transmission (and AR) amplitudes and phases are[37]

$$\frac{d|T_{ee}|}{dl} = \alpha' |T_{ee}|(1 - |T_{ee}|^2) \quad \text{and} \quad \frac{d\phi}{dl} = 0$$

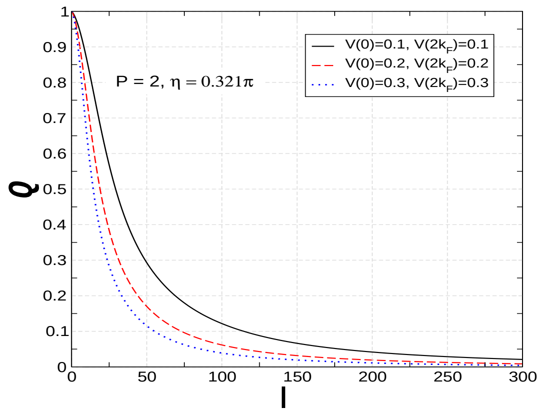

FIG. 4: Pumped charge $\mathcal{Q}$, for pumping in $\Delta_1 - \Delta_2$ plane, is shown as a function of the dimensionless parameter l where $l = ln(L_P/d)$ and L_P is either $L_T = \hbar v_F/k_B T$ at zero bias or $L_V = \hbar v_F/eV$ at zero temperature and d is the short distance cut-off for the RG flow. The three curves correspond to three different values of $V(0)$ and $V(2k_F)$.

$$\frac{d|R_{eh}|}{dl} = -\frac{\alpha'}{2}|R_{eh}|[1-|R_{eh}|^2-|T_{ee}|^2\cos 2(\phi-\theta)]$$
$$\frac{d\theta}{dl} = \frac{\alpha'}{2}|T_{ee}|^2\sin 2(\phi-\theta) \qquad (21)$$

Here $l = ln(L_P/d)$ where d is the short distance cut-off for the RG flow and we have considered the fully symmetric case, *i.e.* $\theta = \theta'$. Unitarity of the S-matrix in Eq. 18 implies that $\phi - \theta = \pi/2 + 2n\pi$ ($n \to$ integer). This simplifies the equations for RG flow for the AR amplitude and phase,

$$\frac{d|R_{eh}|}{dl} = -\alpha'|R_{eh}|\left(1-|R_{eh}|^2\right) \quad \text{and} \quad \frac{d\theta}{dl} = 0 \quad (22)$$

Here, $\alpha' = (g_2+g_1)/2\pi\hbar v_F$ where g_1, g_2 are the running coupling constants whose bare values are set by $g_1(L_P = d) = V(2k_F)$ and $g_2(L_P = d) = V(0)$; $V(x)$ being the inter-electron interaction potential. We now integrate the RG equation for T_{ee} complimented by the RG flow of g_1 and g_2[37] to obtain the L_P dependence of T_{ee} as

$$T_{ee}(L_P) = \frac{T^0_{ee}\left[\left(1+2\alpha_1\ln\frac{L_P}{d}\right)^{\frac{3}{2}}\left(\frac{d}{L_P}\right)^{-(2\alpha_2-\alpha_1)}\right]}{R^0_{eh}+T^0_{ee}\left[\left(1+2\alpha_1\ln\frac{L_P}{d}\right)^{\frac{3}{2}}\left(\frac{d}{L_P}\right)^{-(2\alpha_2-\alpha_1)}\right]} \qquad (23)$$

Here T^0_{ee} and R^0_{eh} are the values of T_{ee} and R_{eh} at length-scale L and $\alpha_1 = V(0)/2\pi\hbar v_F$ and $\alpha_2 = V(2k_F)/2\pi\hbar v_F$. There are two points worth mentioning here : *(a)* the transmission increases with increasing L_P which is a consequence of the fact that the proximity effect due to superconductor induces an effective attractive interaction between the electrons, hence rendering the (Andreev) back-scattering an irrelevant operator, and *(b)* the expression for $T_{ee}(L_P)$ is not in the form of a pure power law even at $T^0_{ee} \to 0$ limit, as is expected from Luttinger Liquid physics because of the RG flow of the g_1, g_2 parameters. Also, it is important to note that we take the short-distance cut-off d to be the distance between the two barriers (L) since this is the length scale at which we glued the two barriers to a single barrier as far as RG is concerned. Using this, we can obtain the scaling behavior of the pumped charge ($\mathcal{Q}$) as a function of the length scale L_P (or the temperature T). In terms of the Landauer−Buttiker conductance, $G_0 = (2e^2/h)\ |T^0_{ee}|^2$, using Eq. 23, we obtain the pumped charge as

$$\mathcal{Q} = \mathcal{Q}_{\rm int} - \left(\frac{d}{L_P}\right)^{-(2\alpha_2-\alpha_1)} \int_0^\tau dt\ I(t)$$

where

$$I(t) = \frac{e}{2\pi}\frac{G_0\left[\left(1+2\alpha_1\ln\frac{L_P}{d}\right)^{\frac{3}{2}}\right]\dot{\delta}}{1+G_0\left[-1+\left(1+2\alpha_1\ln\frac{L_P}{d}\right)^{\frac{3}{2}}\left(\frac{d}{L_P}\right)^{-(2\alpha_2-\alpha_1)}\right]} \qquad (24)$$

Here $\delta = \theta + \phi$ and as earlier, G_0 is expressed in unit of $(2e^2/h)$. $\mathcal{Q}_{\rm int}$ is the integer contribution of the first term in Eq. 20.

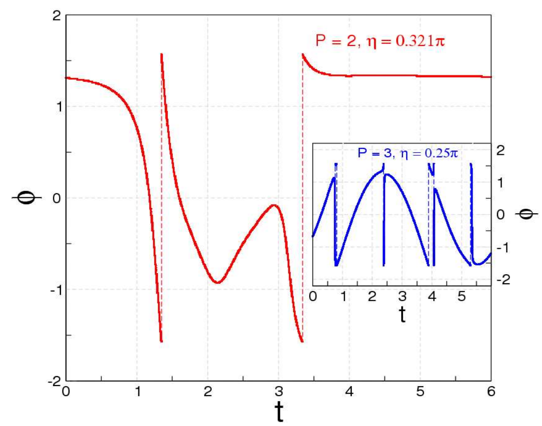

FIG. 5: The plot shows the variation of the AR phase ϕ with time t, along the pumping contour C_1 in the plane of $\Delta_1 - \Delta_2$ and the inset shows the variation of the same along the pumping contour C_2 in the plane of $\phi_1 - \phi_2$.

V. RESULTS AND DISCUSSIONS

1. Pumping in the $\Delta_1 - \Delta_2$ plane : Here the pumped charge is obtained by periodically varying the top gate voltage which controls the Fermi energy of the electrons in the superconducting region. Hence it amounts to varying E/Δ for the two barriers periodically. Just like the double barrier problem, in this case too we observe resonant transmission of electrons at discrete values of E/Δ for fixed values of ϕ_1 and ϕ_2. These discrete values correspond

to the existence of quasi-bound states formed inside the SDB which are quite different from their normal double barrier counterpart as they are produced due to superposition of both electron and hole states and not just any one of them. In Fig. 2 (left panel), we see sharp resonance in transmission probability ($|T_{ee}|^2$) in the $\Delta_1 - \Delta_2$ plane for $L = 1$. We employ the solutions to the RG equations (Eq. 21) to obtain the renormalized surface of transmission in the plane of $\Delta_1 - \Delta_2$ for a value of $L = 10$, this is shown in Fig. 2 (right panel). Note that the RG flow is such that the transmission increases in the entire $\Delta_1 - \Delta_2$ plane, hence reducing the sharpness of resonance and resulting in an increase of transmission (conductance) on the pumping contour C_1 giving rise to reduction in the pumped charge from its quantized value (see Fig. 4). From Fig. 5, we notice that the AR phase ϕ shows a total drop in its value by a factor of 2π during its time evolution along the contour C_1. This drop corresponds to the quantization of the topological part in the expression for pumped charge $\mathcal{Q}$ (Eq. 20) to the value of e.

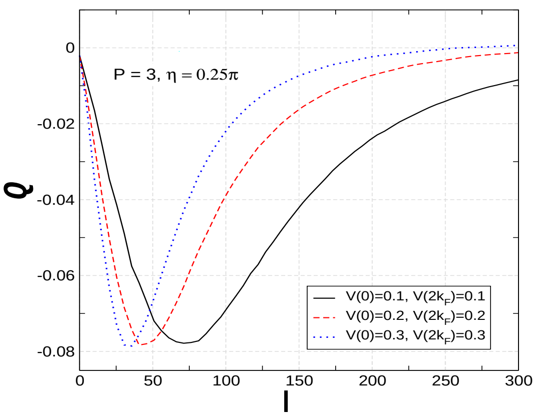

FIG. 6: Pumped charge $\mathcal{Q}$, for pumping in $\phi_1 - \phi_2$ plane is shown in the figure as a function of the dimensionless parameter l where $l = ln(L_P/d)$ and L_P is either $L_T = \hbar v_F/k_B T$ at zero bias or $L_V = \hbar v_F/eV$ at zero temperature and d is the short distance cut-off for the RG flow. The three curves correspond to three different values of $V(0)$ and $V(2k_F)$.

2. Pumping in the $\phi_1 - \phi_2$ plane : In contrast to the previous case, here we obtain two sharp lines of resonances for the transmission function in the $\phi_1 - \phi_2$ plane. Again we observe in Fig. 3 that the RG flow (Eq. 21) results in reduction of the sharpness of the resonance. We consider a pumping contour C_2 which encloses parts of both the resonance lines in the $\phi_1 - \phi_2$ plane. The intersection of the pumping contour C_2 with the lines of resonance results in vanishing of the topological part. This can be seen by observing the time-evolution of the AR phase along contour C_2 as shown in the inset of Fig. 5. In this case the drops are exactly compensated by corresponding rises in phase ϕ by same amount, leading to a net zero topological contribution to the pumped charge. Hence for small values of L_P (see Fig. 6), the pumped charge is almost zero. This is because the topological part is identically zero while the dissipative part is non-zero but vanishingly small (due to the resonance being very sharp) as the conductance on most part of the contour is negligible. As we go to the larger L_P values, the pumped charge shows an interesting non-monotonic behavior, purely coming due to the variation of the dissipative part.

In conclusion, we show that pumping in the $\Delta_1 - \Delta_2$ plane is much more efficient as opposed to that in $\phi_1 - \phi_2$ plane. We also demonstrate that the quantization of the pumped charge is lost in $\Delta_1 - \Delta_2$ plane if we include correlations due to proximity effects in the 1–D QW. Although if the barriers are reflecting then according to RG, the system will flow to the disconnected fixed point ($r = 1$) at low temperature. In that case the sharp transmission resonance would appear in the parameter plane of back-scattering strength of the first and the second barriers. If the pumping contour encloses the transmission resonance, then in the zero temperature limit, the dissipative part of the pumped charge will become vanishingly small resulting in quantized pumped charge. So for the SDB system with small normal reflection, pumped charge will eventually converge to a quantized value in the zero temperature limit.

Acknowledgements

We thank Sumathi Rao for many stimulating and useful discussions and encouragement. We acknowledge use of the Bewoulf cluster at HRI. The work of SD was supported by the Feinberg Fellowship Programme at Weizmann, Israel.

* Email: arijit@hri.res.in
† Email: sourin.das@weizmann.ac.il
[1] P. W. Brouwer, Phys. Rev. B **58**, R10135 (1998).
[2] M. Büttiker, H. Thomas, and A. Pretre, Z. Phys. B **94**, 133 (1994).
[3] M. Switkes, C. M. Marcus, K. Campman, and A. C. Gossard, Science **283**, 1905 (1999).
[4] P. J. Leek *et al*, Phys. Rev. Lett. **95**, 256802 (2005).
[5] M. R. Buitelaar *et al*, *Charge pumping in carbon nanotube quantum dot* (2008), arXiv:0804.3219 [cond-mat].
[6] P. W. Brouwer, A. Lamacraft, and K. Flensberg, Phys. Rev. B **72**, 075316 (2005).

[7] P. Sharma and C. Chamon, Phys. Rev. B **68**, 035321 (2003).
[8] J. Splettstoesser, M. Governale, J. König, and R. Fazio, Phys. Rev. Lett. **95**, 246803 (2005).
[9] E. Sela and Y. Oreg, Phys. Rev. Lett. **96**, 166802 (2006).
[10] D. Fioretto and A. Silva, *Pumping through interacting quantum dots: inelastic scattering and corrections to brouwer's formula* (2007), arXiv:0707.3338 [cond-mat].
[11] R. Citro, N. Andrei, and Q. Niu, Phys. Rev. B **68**, 165312 (2003).
[12] A. Agarwal and D. Sen, Phys. Rev. B **76**, 035308 (2007).
[13] A. F. Morpurgo and F. Beltram, Phys. Rev. B **50**, 1325 (1994).
[14] A. Banerjee, S. Das, and S. Rao, *Quantised charge pumping through multiple quantum dots* (2003), cond-mat/0307324.
[15] Entin-Wohlman and A. Aharony, Phys. Rev. B **66**, 035329 (2002).
[16] S. Banerjee, A. Mukherjee, S. Rao, and A. Saha, Phys. Rev. B **75**, 153407 (2007).
[17] I. L. Aleiner and A. V. Andreev, Phys. Rev. Lett. **81**, 1286 (1998).
[18] Y. Levinson, O. Entin-Wohlman, and P. Wolfle, *Pumping at resonant transmission and transferred charge quantization* (2000), cond-mat/0010494.
[19] S. Das and S. Rao, Phys. Rev. B **70**, 155420 (2004).
[20] D. Yue, L. I. Glazman, and K. A. Matveev, Phys. Rev. B **49**, 1966 (1994).
[21] J. Wang, Y. Wei, B. Wang, and H. Guo, Appl. Phys. Lett. **79**, 3977 (2001).
[22] F.Zhou, Int. J. Mod. Phys. B **15**, 117 (2001).
[23] M. Blaauboer, Phys. Rev. B **65**, 235318 (2002).
[24] M. Governale, F. Taddei, R. Fazio, and F. W. J. Hekking, Phys. Rev. Lett. **95**, 256801 (2005).
[25] F. Taddei, M. Governale, and R. Fazio, Phys. Rev. B **70**, 052510 (2004).
[26] J. Splettstoesser, M. Governale, J. König, F. Taddei, and R. Fazio, Phys. Rev. B **75**, 235302 (2007).
[27] N. Kopnin, A. S. Melnikov, and V. M. Vinokur, Phys. Rev. Lett. **96**, 146802 (2006).
[28] B. Wang and J. Wang, Phys. Rev. B **66**, 201305 (2002).
[29] B. Wang and J. Wang, Phys. Rev. B **65**, 153311 (2002).
[30] S. Russo, J. Tobiska, T. M. Klapwijk, and A. F. Morpurgo, Phys. Rev. Lett. **99**, 086601 (2007).
[31] P. G. de Gennes, *Superconducitivity of Metals and Alloys,* (Addison-Wesley Publishing Co., New York, 1989).
[32] G. E. Blonder, M. Tinkham, and T. M. Klapwijk, Phys. Rev. B **25**, 4515 (1982).
[33] S. Datta, *Electronic transport in mesoscopic systems,* (Cambridge University Press, Cambridge, 1995).
[34] S. Lal, S. Rao, and D. Sen, Phys. Rev. B **66**, 165327 (2002).
[35] S. Das, S. Rao, and D. Sen, Phys. Rev. B **70**, 085318 (2004).
[36] T. Takane and Y. Koyama, J. Phys. Soc. Jpn. **66**, 419 (1997).
[37] S. Das, S. Rao, and A. Saha, Phys. Rev. B **77**, 155418 (2008).
[38] J. Sólyom, Adv. Phys. **28**, 201 (1979).
[39] P. W. Anderson, J. Phys. C **3**, 2436 (1970).